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Thermal wave propagation in a bi-layered composite sphere due to a sudden temperature change on the outer surface

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Abstract

The dynamic thermal behavior of a bi-layered composite sphere due to a sudden temperature change on the outer surface is investigated. The analytical–numerical technique, which is based on the Laplace transformation and the Riemann-sum approximation, is employed to predict the temperature and heat flux histories in the composite sphere. The effects of different parameters such as the relaxation time, the thermal diffusivity ratio, the thermal conductivity ratio, the relaxation time ratio, and the radius ratio of the inner and outer layers of the composite sphere are studied and presented.

1. Introduction

The relation between heat flux and temperature gradient is given by the Fourier law, a parabolic heat conduction model [1,2]. This model is appropriate for many practical heat conduction conditions. In the Fourier law, there is no time difference between the heat flux and temperature gradient, and this implies that the heat propagation speed is infinite.

While the heat transfer situations include extremely high temperature gradients, temperatures near absolute zero, extremely large heat fluxes, and extremely short transient duration, the heat propagation speed is finite, and the Fourier law should be modified [3–11]. Bertman and Sandiford [3] illustrated the wave behavior for heat propagation through helium(II) at 1 K in their experiments. Qiu et al. [4] used femtosecond laser to heat gold films, and found that the experimental results deviated from the Fourier law significantly. Kim et al. [7] used a thermal wave model proposed by Cattaneo and Vernotte to analyze the extreme short-time temperature response of a semi-infinite region due to axisymmetric continuous or pulsed surface heat sources. Al-Nimr and Hader [8] investigated the effect of the phase-lag concept by using the wave theory of heat conduction on the thermal behavior of melting and solidification. Chen and Beraun [9] used the dual-hyperbolic two-step radiation heating model to study ultrashort laser pulse interactions with metal films. Tzou and Chiu [10,11] employed the dual-phase-lag (DPL) model to analyze the thermal response of metal films due to ultrafast laser heating. According to the studies of Kaminski and coworkers [12–14], the propagation speeds of thermal wave in many homogenous and nonhomogenous materials are indeed finite.

The shapes of carbonaceous particles, fuel droplets, molecules, and cells are nearly spherical, can be considered as spherical medium. There are many researches about spherical medium by using the Fourier law [15–18]. When the relaxation time presents for some situations, such as extremely high temperature gradients, temperatures near absolute zero, extremely high heat fluxes, and extremely short transient duration, the Fourier law is no more valid, and the problems should be analyzed by using the thermal wave model. The thermal wave model can be written in the form [6–9,12,19–21]

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Nomenclature

A	coefficient in Eq. (14a)
B	coefficient in Eq. (14b)
c	heat propagation speed [m s^{-1}]
C	coefficient in Eq. (16a)
C_{11}	coefficient in Eq. (15a)
C_{12}	coefficient in Eq. (15b)
C_{13}	coefficient in Eq. (18a)
C_{14}	coefficient in Eq. (18b)
C_{21}	coefficient in Eq. (15c)
C_{22}	coefficient in Eq. (15d)
C_{23}	coefficient in Eq. (18c)
C_{24}	coefficient in Eq. (18d)
D	coefficient in Eq. (16b)
E	coefficient in Eq. (19a)
F	coefficient in Eq. (19b)
i	complex number, $\mathrm{i} = \sqrt{-1}$
k	thermal conductivity [W m^{-1} K^{-1}]
n	parameter in Eq. (20)
N	parameter in Eq. (20)
q	heat flux in the composite sphere [W m^{-2}]
Q	dimensionless heat flux
Q_0	dimensionless heat flux on the outer surface of the composite sphere
$\overline{Q}$	Laplace transform of dimensionless heat flux
$\overline{Q}_{1p}$	the dimensionless heat flux in the inner layer obtained by solving parabolic heat conduction model
$\overline{Q}_{2p}$	the dimensionless heat flux in the outer layer obtained by solving parabolic heat conduction model
Q_n	normalized heat flux
r	radial coordinate [m]
s	Laplacian parameter
t	physical time [s]
T	temperature [K]
T_0	the initial temperature [K]
T_w	the temperature on the outer surface of the composite sphere [K]

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
γ	parameter in Eq. (22)
ε	dimensionless relaxation time
η	dimensionless time
θ	dimensionless temperature
$\bar{\theta}$	Laplace transform of dimensionless temperature
$\bar{\theta}_{1p}$	the dimensionless temperature in the inner layer obtained by solving parabolic heat conduction model
$\bar{\theta}_{2p}$	the dimensionless temperature in the outer layer obtained by solving parabolic heat conduction model
ξ	dimensionless radial coordinate
ξ_r	$\xi_r = \frac{r_1}{r_2}$
τ	relaxation time [s]

Subscripts

1	the inner layer of the composite sphere
2	the outer layer of the composite sphere
r	the ratio of the inner layer to the outer layer of the composite sphere

$$\vec{q}(\vec{r},t) + \tau \frac{\partial \vec{q}(\vec{r},t)}{\partial t} = -k\nabla \vec{T}(\vec{r},t) \tag{1}$$

$$\nabla^2 \vec{T}(\vec{r},t) + \tau \frac{\partial [\nabla^2 \vec{T}(\vec{r},t)]}{\partial t} = \frac{1}{\alpha}\frac{\partial \vec{T}(\vec{r},t)}{\partial t} \tag{2}$$

where $\vec{r}$ is the position vector, t is the physical time, $\vec{q}(\vec{r},t)$ is the heat flux vector, $\vec{T}(\vec{r},t)$ is the temperature distribution field, k is the thermal conductivity, α is the thermal diffusivity, and the relaxation time τ is given as

$$\tau = \frac{\alpha}{c^2} \tag{3}$$

where c is the thermal wave propagation speed. When the thermal wave propagation speed c approaches infinity, the relaxation time τ decreases to zero, and the thermal wave model will reduce to the Fourier law.

Experimental facts showed that the thermal propagation speed is finite in low-pressure gases, cryogenic engineering, nuclear engineering, and seismology [12,21,22]. The dynamic thermal response of a spherical medium is very important in cryogenic engineering and nuclear engineering. Zhang and Liu [14] analyzed the thermal wave propagation in a spherical medium due to a temperature on its surface. Jiang and Liu [22] considered the hyperbolic heat conduction in a hollow spherical medium due to temperatures on its inner surface and outer surfaces. To the authors' knowledge, the dynamic thermal behavior of a bi-layered composite spherical medium has not been investigated yet.

A Laplace transformation and the Riemann-sum approximation method [9,20,23] are used to obtain the solution for the temperature and heat flux histories in the composite sphere.

2. Problems formulations and analysis

The composite sphere is made up of bi-layers of different substances, as depicted in Fig. 1. Initially, the composite sphere is maintained at a uniform tempera-

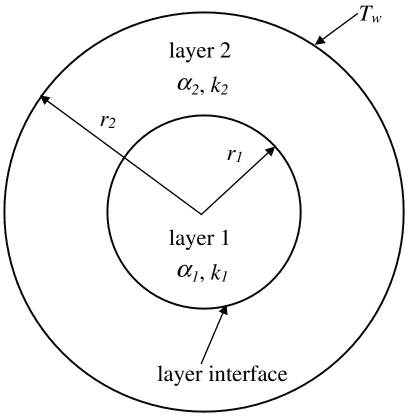

Fig. 1. Schematic diagram of the bi-layered composite sphere.

ture, and there is a sudden temperature change on its outer surface. Due to the high-rate of change of temperature change or temperature field on the outer surface near absolute zero, the thermal wave equation or the hyperbolic heat conduction equation is used to analyze this problem. We assume that the physical and thermal properties of the composite sphere are constant, and there is no contact thermal resistance between the interface of bi-layered substances. The heat flux will start to propagate after some time, the relaxation time τ, when the temperature field has been imposed on the outer surface. The governing equations for this problem are

$$\frac{\partial T_1}{\partial t} = -\frac{\alpha_1}{k_1}\left[\frac{2}{r}q_1 + \frac{\partial q_1}{\partial r}\right], \quad 0 \leqslant r < r_1 \tag{4a}$$

$$\frac{\partial T_1}{\partial r} = -\frac{1}{k_1}\left[q_1 + \tau_1\frac{\partial q_1}{\partial t}\right], \quad 0 \leqslant r < r_1 \tag{4b}$$

$$\frac{\partial T_2}{\partial t} = -\frac{\alpha_2}{k_2}\left[\frac{2}{r}q_2 + \frac{\partial q_2}{\partial r}\right], \quad r_1 \leqslant r \leqslant r_2 \tag{4c}$$

$$\frac{\partial T_2}{\partial r} = -\frac{1}{k_2}\left[q_2 + \tau_2\frac{\partial q_2}{\partial t}\right], \quad r_1 \leqslant r \leqslant r_2 \tag{4d}$$

where subscript 1 represents the properties of the inner layer substances, and subscript 2 the outer layer substances, respectively.

The boundary conditions for $t > 0$ are

$$\frac{\partial T_1(0,t)}{\partial r} = 0 \tag{5a}$$

$$T_2(r_2,t) = T_w \tag{5b}$$

$$T_1(r_1,t) = T_2(r_1,t) \tag{5c}$$

$$q_1(r_1,t) = q_2(r_1,t) \tag{5d}$$

where T_w is the imposed temperature on the outer surface.

The initial conditions at $t = 0$ are

$$T_1(r,0) = T_0, \quad 0 \leqslant r < r_1 \tag{6a}$$

$$T_2(r,0) = T_0, \quad r_1 \leqslant r \leqslant r_2 \tag{6b}$$

$$q_1(r,0) = 0, \quad 0 \leqslant r < r_1 \tag{6c}$$

$$q_2(r,0) = 0, \quad r_1 \leqslant r \leqslant r_2 \tag{6d}$$

where T_0 is the initial temperature of the composite sphere.

In Eq. (4a), $r = 0$ is a singular point, and the conditions for $r = 0$ are

$$q_1(0,t) = 0, \quad r = 0 \tag{7a}$$

$$\lim_{r\to 0^+}\left(\frac{\partial T_1}{\partial r}\right) = 0 \tag{7b}$$

For convenience in the subsequent analysis, we define the following dimensionless parameters:

$$\theta_1(\xi,\eta) = \frac{T_1(r,t) - T_0}{T_w - T_0}, \quad \theta_2(\xi,\eta) = \frac{T_2(r,t) - T_0}{T_w - T_0},$$
$$\xi = \frac{r}{r_2}, \quad \eta = \frac{\alpha_2 t}{r_2^2}, \quad \varepsilon = \frac{\alpha_2\tau_2}{r_2^2}, \quad k_r = \frac{k_1}{k_2}, \quad \alpha_r = \frac{\alpha_1}{\alpha_2},$$
$$\tau_r = \frac{\tau_1}{\tau_2}, \quad Q_1 = \frac{q_1 r_2}{k_2(T_w - T_0)}, \quad Q_2 = \frac{q_2 r_2}{k_2(T_w - T_0)}, \tag{8}$$

The Laplace transform technique is employed to deal with the η-derivative terms in the equations. The Laplace transform of dimensionless temperature $\bar{\theta}$ is defined as

$$L[\theta(\xi,\eta)] = \bar{\theta}(\xi,s) = \int_0^\infty \theta(\xi,\eta)\mathrm{e}^{-s\eta}\,\mathrm{d}\eta \tag{9}$$

where s is the Laplace transform parameter.

Taking the Laplace transform of dimensionless form of Eqs. (4)–(7), we have

$$\overline{Q}_1(0,s) = 0 \tag{10a}$$

$$\lim_{\xi\to 0^+}\left(\frac{\mathrm{d}\bar{\theta}_1}{\mathrm{d}\xi}\right) = 0 \tag{10b}$$

$$s\bar{\theta}_1 = -\frac{\alpha_r}{k_r}\left[\frac{2}{\xi}\overline{Q}_1 + \frac{\mathrm{d}\overline{Q}_1}{\mathrm{d}\xi}\right], \quad 0 < \xi < \xi_r \tag{10c}$$

$$\frac{\mathrm{d}\bar{\theta}_1}{\mathrm{d}\xi} = -\frac{1}{k_r}\left[\overline{Q}_1 + \tau_r\varepsilon s\overline{Q}_1\right], \quad 0 < \xi < \xi_r \tag{10d}$$

$$s\bar{\theta}_2 = -\left[\frac{2}{\xi}\overline{Q}_2 + \frac{\mathrm{d}\overline{Q}_2}{\mathrm{d}\xi}\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{10e}$$

$$\frac{\mathrm{d}\bar{\theta}_2}{\mathrm{d}\xi} = -\left[\overline{Q}_2 + \varepsilon s\overline{Q}_2\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{10f}$$

$$\frac{\mathrm{d}\bar{\theta}_1(0,s)}{\mathrm{d}\xi} = 0 \tag{11a}$$

$$\bar{\theta}_2(1,s) = \frac{1}{s} \tag{11b}$$

$$\bar{\theta}_1(\xi_r,s) = \bar{\theta}_2(\xi_r,s) \tag{11c}$$

$$\overline{Q}_1(\xi_r,s) = \overline{Q}_2(\xi_r,s) \tag{11d}$$

$$\bar{\theta}_1(\xi,0) = 0, \quad 0 < \xi < \xi_r \tag{12a}$$

$$\bar{\theta}_2(\xi,0) = 0, \quad \xi_r \leqslant \xi \leqslant 1 \tag{12b}$$

$$\overline{Q}_1(\xi,0) = 0, \quad 0 < \xi < \xi_r \tag{12c}$$

$$\overline{Q}_2(\xi,0) = 0, \quad \xi_r \leqslant \xi \leqslant 1 \tag{12d}$$

where $\xi_r = \frac{r_1}{r_2}$.

The solutions to Eqs. (10a)–(10f) subject to Eqs. (11a)–(11d) and (12a)–(12d) are

$$\lim_{\xi\to 0^+} \bar{\theta}(\xi,s) = \text{finite constant} \tag{13a}$$

$$\bar{\theta}_1(\xi,s) = \frac{1}{\xi}\left[C_{11}e^{\sqrt{As}\xi} + C_{12}e^{-\sqrt{As}\xi}\right], \quad 0 < \xi < \xi_r \tag{13b}$$

$$\bar{\theta}_2(\xi,s) = \frac{1}{\xi}\left[C_{21}e^{\sqrt{Bs}\xi} + C_{22}e^{-\sqrt{Bs}\xi}\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{13c}$$

$$\overline{Q}_1(0,s) = 0 \tag{13d}$$

$$\overline{Q}_1(\xi,s) = -\frac{k_r}{\alpha_r A}\left[\frac{(\sqrt{As}\xi - 1)C_{11}e^{\sqrt{As}\xi} - (\sqrt{As}\xi + 1)C_{12}e^{-\sqrt{As}}\xi}{\xi^2}\right], \quad 0<\xi<\xi_r \tag{13e}$$

$$\overline{Q}_2(\xi,s) = -\frac{1}{B}\left[\frac{(\sqrt{Bs}\xi - 1)C_{21}e^{\sqrt{Bs}\xi} - (\sqrt{Bs}\xi + 1)C_{22}e^{-\sqrt{Bs}}\xi}{\xi^2}\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{13f}$$

where

$$A = \frac{1 + \tau_r \varepsilon s}{\alpha_r} \tag{14a}$$

$$B = 1 + \varepsilon s \tag{14b}$$

$$C_{11} = \frac{C_{21}e^{\sqrt{Bs}\xi_r} + C_{22}e^{-\sqrt{Bs}\xi_r}}{e^{\sqrt{As}\xi_r} - e^{-\sqrt{As}\xi_r}} \tag{15a}$$

$$C_{12} = -C_{11} \tag{15b}$$

$$C_{21} = \frac{(\sqrt{Bs}\xi_r + 1 + C)e^{-\sqrt{Bs}\xi_r}}{sD} \tag{15c}$$

$$C_{22} = \frac{(\sqrt{Bs}\xi_r - 1 - C)e^{\sqrt{Bs}\xi_r}}{sD} \tag{15d}$$

in which

$$C = \frac{k_r B[(\sqrt{As}\xi_r - 1)e^{\sqrt{As}\xi_r} + (\sqrt{As}\xi_r + 1)e^{-\sqrt{As}\xi_r}]}{\alpha_r A\left[e^{\sqrt{As}\xi_r} - e^{-\sqrt{As}\xi_r}\right]} \tag{16a}$$

$$D = (\sqrt{Bs}\xi_r - 1 - C)e^{(\xi_r - 1)\sqrt{Bs}} + (\sqrt{Bs}\xi_r + 1 + C)e^{(1-\xi_r)\sqrt{Bs}} \tag{16b}$$

When the thermal propagation speed c is infinite, the relaxation time τ is zero. The governing equations (4a)–(4d) will be reduced to the classical Fourier law, the parabolic heat conduction equation, whose solutions are

$$\lim_{\xi\to 0^+} \bar{\theta}_{1p}(\xi,s) = \text{finite constant} \tag{17a}$$

$$\bar{\theta}_{1p}(\xi,s) = \frac{1}{\xi}\left[C_{13}e^{\sqrt{\frac{s}{\alpha_r}}\xi} + C_{14}e^{-\sqrt{\frac{s}{\alpha_r}}\xi}\right], \quad 0 \leqslant \xi \leqslant \xi_r \tag{17b}$$

$$\bar{\theta}_{2p}(\xi,s) = \frac{1}{\xi}\left[C_{23}e^{\sqrt{s}\xi} + C_{24}e^{-\sqrt{s}\xi}\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{17c}$$

$$\overline{Q}_{1p}(0,s) = 0 \tag{17d}$$

$$\overline{Q}_{1p}(\xi,s) = -k_r\left[\frac{\left(\sqrt{\frac{s}{\alpha_r}}\xi - 1\right)C_{13}e^{\sqrt{\frac{s}{\alpha_r}}\xi} - \left(\sqrt{\frac{s}{\alpha_r}}\xi + 1\right)C_{14}e^{-\sqrt{\frac{s}{\alpha_r}}\xi}}{\xi^2}\right], \quad 0<\xi<\xi_r \tag{17e}$$

$$\overline{Q}_{2p}(\xi,s) = -\left[\frac{(\sqrt{s}\xi - 1)C_{23}e^{\sqrt{s}\xi} - (\sqrt{s}\xi + 1)C_{24}e^{-\sqrt{s}\xi}}{\xi^2}\right], \quad \xi_r \leqslant \xi \leqslant 1 \tag{17f}$$

$$C_{13} = \frac{C_{23}e^{\sqrt{s}\xi_r} + C_{24}e^{-\sqrt{s}\xi_r}}{e^{\sqrt{\frac{s}{\alpha_r}}\xi_r} - e^{-\sqrt{\frac{s}{\alpha_r}}\xi_r}} \tag{18a}$$

$$C_{14} = -C_{13} \tag{18b}$$

$$C_{23} = \frac{(\sqrt{s}\xi_r + 1 + E)e^{-\sqrt{s}\xi_r}}{sF} \tag{18c}$$

$$C_{24} = \frac{(\sqrt{s}\xi_r - 1 - E)e^{\sqrt{s}\xi_r}}{sF} \tag{18d}$$

in which

$$E = \frac{k_r\left[\left(\sqrt{\frac{s}{\alpha_r}}\xi_r - 1\right)e^{\sqrt{\frac{s}{\alpha_r}}\xi_r} + \left(\sqrt{\frac{s}{\alpha_r}}\xi_r + 1\right)e^{-\sqrt{\frac{s}{\alpha_r}}\xi_r}\right]}{e^{\sqrt{\frac{s}{\alpha_r}}\xi_r} - e^{-\sqrt{\frac{s}{\alpha_r}}\xi_r}} \tag{19a}$$

$$F = (\sqrt{s}\xi_r - 1 - E)e^{(\xi_r - 1)\sqrt{s}} + (\sqrt{s}\xi_r + 1 + E)e^{(1-\xi_r)}\sqrt{s} \tag{19b}$$

3. Analytical–numerical technique

To obtain the temperature and heat flux histories of the bi-layered composite sphere, Eqs. (13a)–(13f) and (17a)–(17f) must be transformed back into the time domain. Bromwich contour integration is the standard procedure and can be used to obtain the inverse solution of Laplace transform of Eqs. (13a)–(13f) and (17a)–(17f). Owing to the complicated integrands usually involved in Bromwich contour integration equations, the Riemann-sum approximation method is used to obtain the inverse solutions of Laplace transform [20,23]

$$\theta(\xi,\eta) \cong \frac{e^{\gamma\eta}}{\eta}\left[\frac{1}{2}\bar{\theta}(\xi,\gamma) + \mathrm{Re}\sum_{n=1}^{N}\bar{\theta}\left(\xi,\gamma+\frac{in\pi}{\eta}\right)(-1)^n\right],$$
$$i = \sqrt{-1} \tag{20}$$

where $\bar{\theta}$ is the dimensionless temperature obtained from Eqs. (13a)–(13f) or (17a)–(17f), Re is the real part of the summation in Eq. (20), the maximum value of N is 4×10^6. The stopping criterion for Eq. (20) is

$$\left|\frac{\theta_N - \theta_{N-1}}{\theta_{N-1}}\right| \leqslant 10^{-7}, \tag{21}$$

and γ is determined from Eq. (22)

$$\gamma\eta \cong 4.7 \tag{22}$$

4. Results and discussions

To demonstrate the validity of the analytical–numerical technique and the solutions of Eqs. (13a)–(13c), the results of the special case of the bi-layered composite sphere are compared with the analytical solution of Zhang and Liu [14]. When $\xi_r = 1$, $\alpha_r = 1$, $k_r = 1$ and $\tau_r = 1$, the bi-layered composite sphere can be considered as a one-layer solid sphere. Fig. 2(a) and (b) indicates that there are good agreements between the analytical solutions of Zhang and Liu [14] and the analytical–numerical solutions of Eqs. (13a)–(13c) for a single-layer sphere. Fig. 2(a) and (b) also shows that the heat propagation speeds are finite.

Fig. 3 shows that the temperature distributions in the bi-layered composite sphere plotted as a function of the radial position for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$. When $\eta = 0.2$, the thermal wave had not reached the interface yet. For $\eta = 0.5$, the thermal wave had passed and left the interface, and it was split into two waves in the opposing direction. The transmitted thermal wave retained the initial wavelike characteristic and continued to travel to the center of the composite sphere. The reflected thermal wave traveled to the outer surface. As Fig. 3 indicated, the layer interface reflected the thermal wave back to the outer surface of the sphere. For $\eta = 0.6$, the thermal wave had reached the center of the composite sphere, the magnitude of the transmitted thermal wave was diminished. When the value of η increases, the waveform is gradually blunted and spread due to the dissipation mechanism. The dimensionless times η for the composite sphere reaching a quasi-steady state are 0.6 and 4.6 predicted by the parabolic and hyperbolic heat conduction models, respectively. Shown in Fig. 3, the results obtained by solving the hyperbolic heat conduction equation demonstrate that the thermal

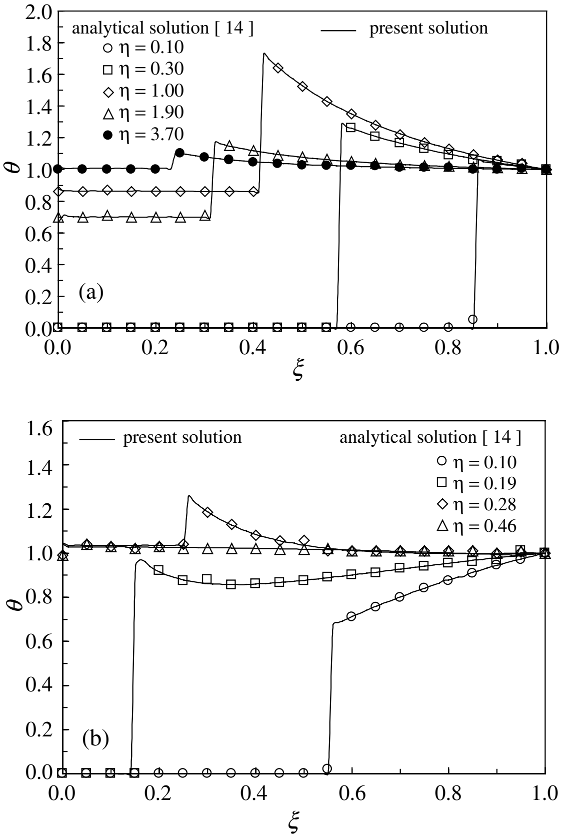

Fig. 2. Comparisons between the analytical solutions of Zhang and Liu [14] and the analytical–numerical solutions of Eqs. (13a)–(13c) for a spherical medium, when $\xi_r = 1$, $\alpha_r = 1$, $k_r = 1$, and $\tau_r = 1$: (a) $\varepsilon = 0.5$, (b) $\varepsilon = 0.05$.

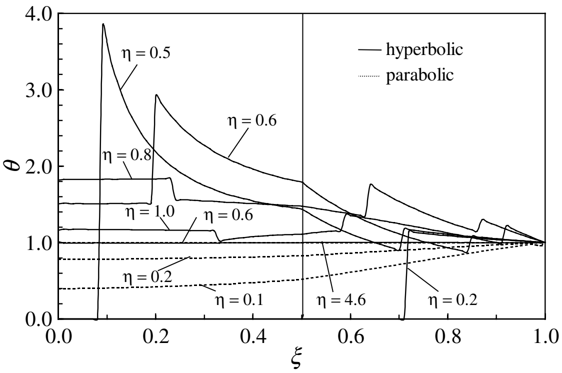

Fig. 3. Temperature distributions in the bi-layered composite sphere plotted as a function of the radial position ξ for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$.

wave front propagates at a finite speed in the composite sphere and the temperature fields become very steep in the vicinities of the thermal wave front. The temperature profiles predicted by the parabolic heat conduction equation are lower than that by the hyperbolic heat conduction equation for the same dimensionless time η.

The effect of dimensionless relaxation time ε on the temperature distribution in the bi-layered composite sphere at dimensionless time $\eta = 0.5$ is shown in Fig. 4. For $\varepsilon = 0.1$, the waveform almost disappeared; for $\varepsilon = 0.5$, the thermal wave had not reached the center of the composite sphere. As expected, the larger the value of the relaxation time ε is, the lower the thermal wave speed is. Fig. 5 reveals that the heat propagation speed varies with the thermal diffusivity ratio α_r. From the definition of relaxation time, the thermal wave speed varies with the thermal diffusivity proportionally. Fig. 5 also displays that the effect of thermal diffusivity ratio α_r on the temperature distributions predicted by the parabolic heat conduction model are not significant as those predicted by the hyperbolic heat conduction model. The effect of thermal conductivity ratio k_r on the heat propagation speed in the bi-layered composite sphere is negligible, as depicted in Fig. 6. The temperature in the composite spherical medium will decrease as the value of k_r increases.

When the value of the relaxation time ratio $\tau_r > 1$, it implies that the heat propagation speed in the inner layer of the composite sphere will be lower than that in the outer layer. As depicted in Fig. 7, the larger the value of the relaxation time ratio τ_r is, the lower the heat propagation speed is. For the case of $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$, the thermal wave speed in the inner layer of the composite sphere will be higher than that in the outer layer. As shown in Fig. 8(a), the larger the value of

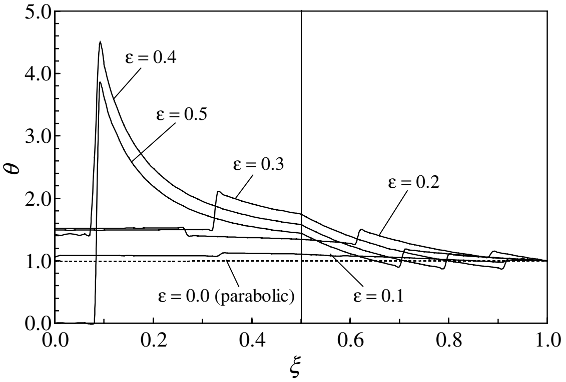

Fig. 4. The effect of dimensionless relaxation time ε on the temperature distribution in the bi-layered composite sphere for $\eta = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$.

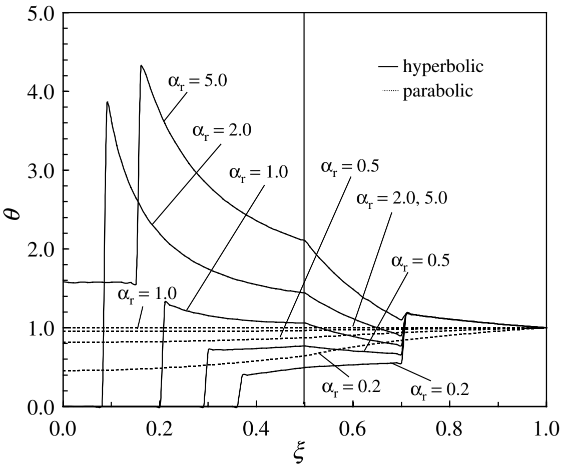

Fig. 5. The effect of thermal diffusivity ratio α_r on the temperature distribution in the bi-layered composite sphere for $\eta = 0.5$, $\xi_r = 0.5$, $k_r = 2.0$, $\tau_r = 0.5$, and $\varepsilon = 0.5$.

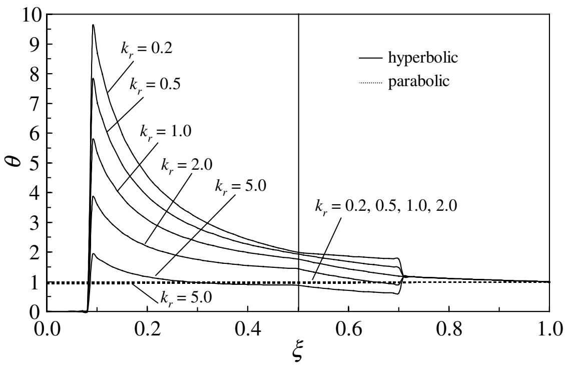

Fig. 6. The effect of thermal conductivity ratio k_r on the temperature distribution in the bi-layered composite sphere for $\eta = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $\tau_r = 0.5$, and $\varepsilon = 0.5$.

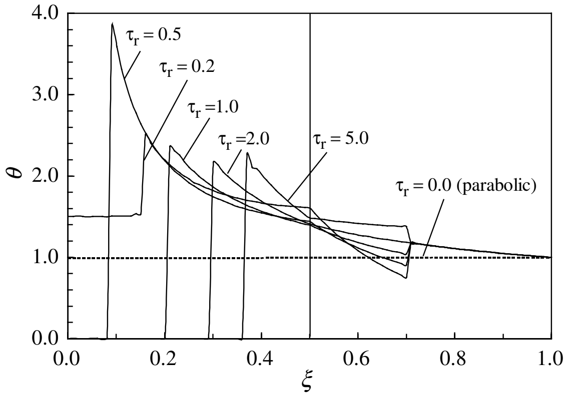

Fig. 7. The effect of dimensionless relaxation time ratio τ_r on the temperature distribution in the bi-layered composite sphere for $\eta = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\varepsilon = 0.5$.

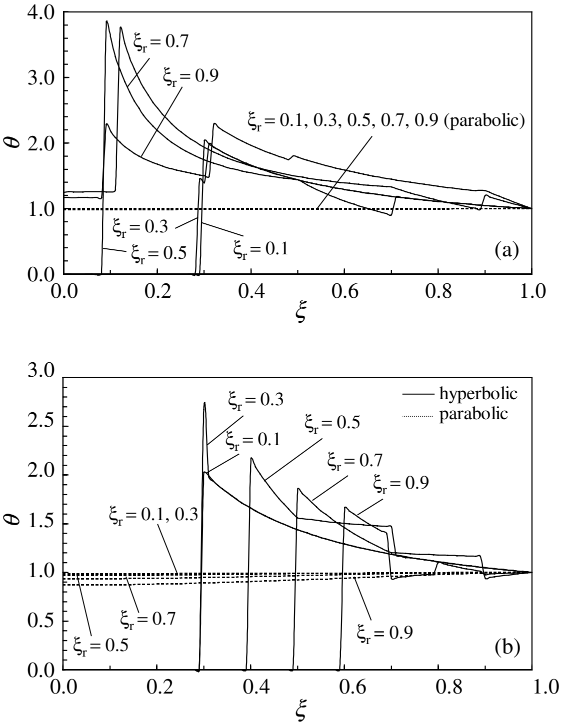

Fig. 8. The effect of the dimensionless radius ratio ξ_r on the temperature distribution in the bi-layered composite sphere for $\eta = 0.5$, and $\varepsilon = 0.5$: (a) $\alpha_r = 2.0$, $k_r = 2.0$, $\tau_r = 0.5$; (b) $\alpha_r = 0.5$, $k_r = 0.5$, $\tau_r = 2.0$.

the relaxation time ratio τ_r is, the higher the heat propagation speed is. On the other hand, when $\alpha_r = 0.5$, $k_r = 0.5$, and $\tau_r = 2.0$, the heat transfer propagation speed in the inner layer of the composite sphere will be lower than that in the outer layer, and the opposite results can be displayed in Fig. 8(b).

In Fig. 9, the dimensionless temperature θ is plotted as a function of the dimensionless time η at the interface of the bi-layered composite sphere with $\xi = 0.5$, for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 0.5$, and $\tau_r = 0.5$. The larger the value of the dimensionless relaxation time ε is, the longer time that the composite sphere possesses a uniform temperature is needed. Fig. 10 shows that the variation of temperature with t/τ at $\xi = 0.5$ for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$. The wave nature of heat propagation will disappear when $t/\tau > 12$, and these results are similar to the solutions obtained by Zhang and Liu [14] for a one-layer sphere.

The normalized heat flux Q_n is defined as

$$Q_n = \frac{Q}{Q_0} \tag{23}$$

where Q is the dimensionless heat flux in the bi-layered composite sphere, and Q_0 is the dimensionless heat flux on the outer surface.

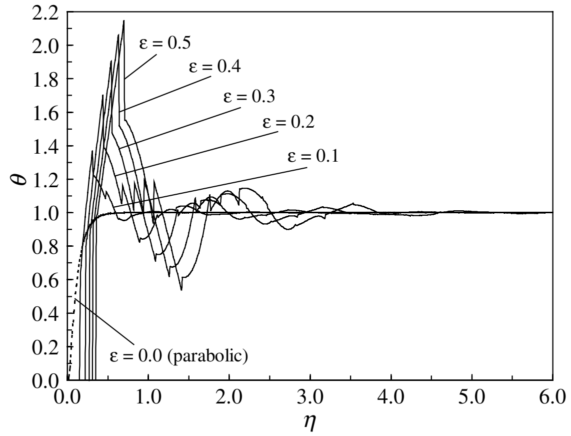

Fig. 9. Predicted temperature at $\xi = 0.5$ plotted as a function of the dimensionless time η for $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$.

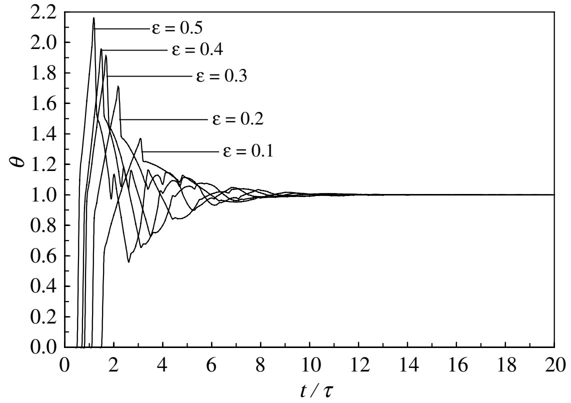

Fig. 10. Predicted temperature at $\xi = 0.5$ plotted as a function of the time ratio t/τ for $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$.

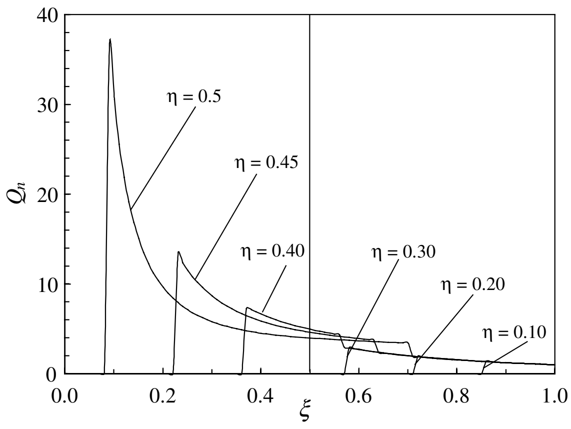

Fig. 11. Predicted heat flux distribution in the bi-layered composite sphere plotted as a function of the radial position ξ for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$.

Fig. 11 displays the heat flux distribution in the bilayered composite sphere as a function of the radial position for $\varepsilon = 0.5$, $\xi_r = 0.5$, $\alpha_r = 2.0$, $k_r = 2.0$, and $\tau_r = 0.5$. When $\eta = 0.1$, 0.2, and 0.3, the thermal wave had not reached the interface of the composite sphere yet. All the heat flux distributions were coincided one another before the thermal wave reached the interface, since they had not been affected by the reflected waves. The heat flux histories predicted by the hyperbolic heat conduction equation also reveals that the heat propagation speed is finite.

5. Conclusions

The dimensionless temperatures predicted by the hyperbolic heat conduction model are greater than the corresponding values for the parabolic heat conduction model. The temperature profiles predicted by the hyperbolic heat conduction equation model in the composite spherical medium are larger than the temperature on the outer surface. These results are different from the solutions predicted by the parabolic heat conduction model. Both the temperature profiles and heat flux histories reveal the wave nature of heat propagation, and that the heat propagation speed is finite.

The effects of different parameters such as the relaxation time ε, the radius ratio ξ_r, the thermal diffusivity ratio α_r, the thermal conductivity ratio k_r, and the relaxation time ratio τ_r of the inner and outer medium are studied and presented. The larger the value α_r is, the higher the heat propagation speed is. The larger the values ε and τ_r are, the lower the thermal speed is. The effect of k_r on the heat propagation speed is negligible. When $t/\tau > 12$, the wave feature of heat propagation will disappear.

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